Mr. Michael T. Davis Calculus	Sections 5.2 & 5.3 Quiz April 21, 2017

Name: Solutions

6 pts 1. Determine whether the function $f(x)=x^2-5x-10$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1,2]$. If it does, then find the value of c that satisfies the equation $f'(c)=\frac{f(b)-f(a)}{b-a}$.

f(x) is cont. on $[-1,2]$ $\quad f'(x)=2x-5$

f(x) is differ. on $(-1,2)$

$$m=\frac{f(2)-f(-1)}{2--1}=\frac{2^2-5(2)-10-((-1)^2-5(-1)-10)}{3}$$

$$m=\frac{4-10-10-1-5+10}{3}=\frac{-12}{3}=-4$$

$2x-5=-4 \quad 2x=1 \quad x=\frac{1}{2} \quad c=\frac{1}{2}$

4 pts 2. Determine whether the function $f(x)=\frac{\sin x}{x}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-\pi,\pi]$. If it does, then find the value of c that satisfies the equation $f'(c)=\frac{f(b)-f(a)}{b-a}$.

f(x) is not continuous on $[-\pi,\pi]$ since $f'(0)=$ undefined

Therefore, the function f(x) does not satisfy hypothesis I of the MVT.

4pts 3. Determine whether the function $f(x)=|x-1|$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0,3]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

$$f'(x)=\begin{cases}-1 & \text{if } x<1\\ 1 & \text{if } x>1\end{cases}$$

$f(x)$ is cont. on $[0,3]$

$f(x)$ is not diff. at $x=1$ since the graph of $f(x)$ has a corner at $x=1$ or since $\lim_{x\to1^-} f'(x) \neq \lim_{x\to1^+} f'(x)$ since $\lim_{x\to1^-} f'(x) = -1$ and $\lim_{x\to1^+} f'(x) = 1$

$f(x)$ does not satisfy hypothesis II of the MVT

7pts 4. Determine the interval(s) on which the function $f(x)=x^2-10x+1$ is (a) increasing and (b) decreasing.

$f'(x) = 2x-10 \quad 0 = 2x-10 \quad x=5 \quad f'(5)=0$

$- - - - \quad + + + +$ f' (sign chart, critical point at 5)

(a) $f(x)$ is increasing on $(5,\infty)$ since $f'(x)>0$

(b) $f(x)$ is decreasing on $(-\infty,5)$ since $f'(x)<0$

7pts 5. Determine the interval(s) on which the function $f(x)=\ln x$ is (a) increasing and (b) decreasing.

$f'(x) = \frac{1}{x} \quad f'(0) = \text{undefined}$

Domain of $f(x)$ is $(0,\infty)$

$- - - - \quad + + + +$ f' (sign chart, 0 marked)

(a) $f(x)$ is increasing on $(0,\infty)$ since $f'(x)>0$

(b) $f(x)$ is decreasing on $(-\infty,0)$ since $f'(x)<0$

7pts 6. Determine the interval(s) on which the function $f(x)=e^{8x-x^2}$ is (a) increasing and (b) decreasing.

$f'(x) = (8-2x)e^{8x-x^2} \quad 0 = (8-2x)e^{8x-x^2} \quad x=4$

$f'(4)=0$

$+ + + + + \quad - - - - - -$ f' (sign chart, critical point at 4)

(a) $f(x)$ is increasing on $(-\infty,4)$ since $f'(x)>0$

(b) $f(x)$ is decreasing on $(4,\infty)$ since $f'(x)<0$

8pt 7. Find all the point(s) of inflection of the function $f(x)=x^3+3x^2-9x+3$

$f'(x) = 3x^2+6x-9$ $f''(x) = 6x+6$ $0 = f''(x)$

$0 = 6x+6$ $x=-1$ $f''(-1)=0$

f'': $- - - -$ on $(-\infty,-1)$; $+ + + +$ on $(-1,\infty)$

f(x) has a point of inflection at $x=-1$ since $f'(-1)=-12$, therefore, f(x) has a tangent at $x=-1$ and since $f''(x)$ changes sign at $x=-1$

8pt 8. Find all the point(s) of inflection of the function $f(x)=\cos x$ on $[0,2\pi]$

$f'(x) = -\sin x$ $f''(x) = -\cos x$ $f''(x)=0$

$0 = -\cos x$ $x=\frac{\pi}{2}$ & $x=\frac{3\pi}{2}$ $f''(\frac{\pi}{2})=0$ & $f''(\frac{3\pi}{2})=0$

f'': $- - -$ on $(0,\frac{\pi}{2})$; $+ + + +$ on $(\frac{\pi}{2},\frac{3\pi}{2})$; $- - -$ on $(\frac{3\pi}{2},2\pi)$

f(x) has points of inflection at $x=\frac{\pi}{2}$ & $x=\frac{3\pi}{2}$ since $f'(\frac{\pi}{2})=-1$ and $f'(\frac{3\pi}{2})=1$ and since $f''(x)$ changes sign at both $x=\frac{\pi}{2}$ and $x=\frac{3\pi}{2}$

9pts 9. Use the Concavity Test to determine the interval(s) on which the graph of the function $f(x)=xe^x$ is (a) concave up and (b) concave down.

$f'(x) = e^x+xe^x$ $f''(x) = e^x+e^x+xe^x = e^x(2+x)$

$f''(-2)=0$ $f'(-2)=\frac{-1}{e^2}$

f'': $- - - -$ on $(-\infty,-2)$; $+ + + +$ on $(-2,\infty)$

(a) f(x) is concave up on $(-2,\infty)$ since $f''>0$

(b) f(x) is concave down on $(-\infty,-2)$ since $f''<0$

9 pts 10. Use the Concavity Test to determine the interval(s) on which the graph of the function $f(x)=\frac{x^4}{12}-\frac{2x^3}{3}-\frac{5x^2}{2}-6x+7$ is (a) concave up and (b) concave down.

$f'(x) = \frac{x^3}{3} - 2x^2-5x-6$ $f''(x) = x^2-4x-5$

$f''(x)=0$ $0=(x-5)(x+1)$ $f''(5)=0$ and $f''(-1)=0$

f'': $+ + +$ on $(-\infty,-1)$; $- - - - -$ on $(-1,5)$; $+ + +$ on $(5,\infty)$

(a) f(x) is concave up on $(-\infty,-1)\cup(5,\infty)$ since $f''(x)>0$

(b) f(x) is concave down on $(-1,5)$ since $f''(x)<0$